except when $u=0$ at the ends of the element.

The correct strain energy term from which the element stiffness matrix should have been derived would therefore be

$$U_e = \frac{1}{2}\int_0^l EIw,_{xx}^2 \mathrm{d}x + \frac{EA}{2L}\left(\int_0^L \frac{1}{2}\bar{w},_{\bar{x}}^2 \mathrm{d}\bar{x}\right)\left(\int_0^l \frac{1}{2}w,_x^2 \mathrm{d}x\right) \quad (8)$$

This raises some subtle philosophical problems. In the finite element method, a structure is discretized using a number of finite elements each having structural properties based on displacement assumptions over the element only and independent of structural properties of the rest of the structure. The entire structural system is then built up by assembling such elements and imposing compatibility of displacements at the inter-element boundaries. However, in the present method, the functional to be minimized, $(T_e - U_e)$ as indicated by Eq. (8) contains both element displacement quantities $w,_x$ over $x=0$ to l and system displacement quantities $\bar{w},_{\bar{x}}$ over $\bar{x}=0$ to L. This therefore violates the basic philosophy of the finite element method.

The conclusion to be derived from this exercise is that it may not be possible to develop a Ritz-type finite element analog for the moderately large bending theory of a beam in terms of the transverse displacement function alone, although it is possible in a continuum approach.

Acknowledgment

The authors have benefited greatly from discussions with H. V. Lakshminarayana, Structures Division, N.A.L. The first author is grateful to the N.A.L. and the C.S.I.R. for the award of a Research Associateship.

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Reply by Author to G. Prathap and G. R. Bhashyam

Chuh Mei*
Old Dominion University, Norfolk, Va.

THE author wishes to thank Dr. Prathap for his interest in the work and for his many comments.[1-5] However, the author does not agree with the conclusion reached in the present Comment. First of all, to find out whether an assumption or a physical model is vaid or not, one should compare results from the model with experiments or other reliable solutions. That was exactly what Prathap and Bhashyam have done by redefining the axial force as an average of element axial forces and comparing the results with continuum solutions. Second, the axial force developed due to large amplitudes, which is treated as constant along the length of the beam, is also an approximation. It is based on an assumption that the longitudinal inertia effects can be neglected. The finite element method[6-8] involves an approximation of the axial force N_e which varies from element to element or an assumption that the axial displacement can be neglected for moderately large amplitude vibrations of a beam with immovable edges. Is this a good and valid assumption? The following clearly gives a positive answer.

1) Figure 3 of Ref. 6 gives a comparison of resonant frequency between the finite element method, three continuum solutions, and experimental data.[9] The finite element approach gives the prediction that agrees best with the experiment.

2) An analytical investigation[10,11] has shown that the effect on nonlinear frequency of inclusion of longitudinal deformation in the strain-displacement relations is not of much significance for moderately large amplitude vibrations of slender beams and thin plates.

3) Furthermore, the finite element results,[6-8,12,13] including the frequency ratios for higher modes (Table 1 of Ref. 6), all were in good agreement with those available in the literature.

In conclusion, the author would like to make a few comments. Since Refs. 6-8 and 12 are the first general formulation by the finite element method for studying the nonlinear vibrational characteristics of beams and thin plates, further modifications are expected to improve upon it. In view of the lack of test data in the literature for large amplitude vibrations, more experimental data are urgently needed.

Received Nov. 13, 1979.
Index category: Structural Dynamics.

*Associate Professor, Dept. of Mechanical Engineering and Mechanics. Member AIAA.

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